Problems

This is a set of drills for practice with series. Try to determine the convergence of these series in more than one way!

1. Determine convergence of given series. When the series converges, determine value to which it converges.

   (a)
   $$\sum_{n=1}^{\infty} \frac{n+1}{2n}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{2^n}{5^{n+1}}$$

   (c)
   $$\sum_{n=1}^{\infty} \cos\left(\frac{n\pi}{2}\right)$$

   (d)
   $$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$$

   (e)
   $$\sum_{n=1}^{\infty} \frac{7^{2n+3}}{3^{2n-2}}$$

   (f)
   $$\sum_{n=1}^{\infty} \frac{7^{n+3}}{3^{2n-2}}$$

   (g)
   $$\sum_{n=1}^{\infty} \sqrt{\frac{n^2-1}{n^2+1}}$$

   (h)
   $$\sum_{n=1}^{\infty} \frac{\cos n\pi}{2^n}$$

   (i)
   $$\sum_{n=1}^{\infty} \frac{4^n+3^n}{3^n}$$

   (j)
   $$\sum_{n=1}^{\infty} \arctan n$$

   (k)
   $$\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n}$$

   (l)
   $$\sum_{n=1}^{\infty} \frac{3^n-1}{2^n}$$

   (m)
   $$\sum_{n=1}^{\infty} \frac{n^2+2^{2n}}{4^n}$$

   (n)
   $$\sum_{n=1}^{\infty} \frac{2^n+4^n-8^n}{2^{3n}}$$

2. Determine whether the given series converges or diverges.

   (a)
   $$\sum_{n=1}^{\infty} \frac{1}{4n-3}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{1}{5n^2-3n-1}$$

   (c)
   $$\sum_{n=4}^{\infty} \frac{n^2}{n^4-6n^2+5}$$

   (d)
   $$\sum_{n=1}^{\infty} \frac{n-5}{n^2+3n-2}$$

   (e)
   $$\sum_{n=1}^{\infty} n^2 e^{-2n}$$

(f)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n+5}}{n^3+3}$$

(g)
$$\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$$

(h)
$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2+1}}{n^3} \arctan n$$

(i)
$$\sum_{n=2}^{\infty} \frac{1+(\ln n)^2}{n(\ln n)^2}$$

(j)
$$\sum_{n=1}^{\infty} \frac{\ln(n+1)}{n+1}$$

(k)
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt[3]{\ln n}}$$

3. Determine whether the given series converges or diverges. (Most of these involve the ratio test.)

(a)
$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

(b)
$$\sum_{n=1}^{\infty} \frac{1}{n^n}$$

(c)
$$\sum_{n=4}^{\infty} \frac{(2n)!}{(n!)^2}$$

(d)
$$\sum_{n=1}^{\infty} \frac{3^{-n}+2^{-n}}{4^{-n}+5^{-n}}$$

(e)
$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot \cdots \cdot (2n)}{4 \cdot 7 \cdot \cdots \cdot (3n+1)}$$

(f)
$$\sum_{n=1}^{\infty} n \left(\frac{3}{4}\right)^n$$

(g)
$$\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot \cdots \cdot (2n)}{3 \cdot 5 \cdot \cdots \cdot (2n+1)} \left(\frac{1}{n^2}\right)$$

(h)
$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{n+1}}$$

(i)
$$\sum_{n=2}^{\infty} \frac{2^n + n^2 3^n}{4^n}$$

(j)
$$\sum_{n=1}^{\infty} \frac{(2n)!}{(3n)!} 5^{2n}$$

4. Determine whether the given series converges absolutely, converges conditionally, or diverges. Remember to consider the series of absolute values when some of the terms are negative.

(a)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$$

(b)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$$

(c)
$$\sum_{n=4}^{\infty} (-1)^n \frac{3^n}{n^3}$$

(d)
$$\sum_{n=1}^{\infty} \frac{n \sin(n\pi/4)}{2^n}$$

(e)
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{3n-2}}{n}$$

(f)
$$\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n^2+3}}{n^2+5}$$

Answers!

1. Answers for section 1
   (b) $\frac{2}{15}$
   (d) diverges
   (f) 10804.5
   (h) $\frac{-1}{3}$
   (j) diverges
   (l) 4
   (n) diverges
2. Answers for section 2
   (a) diverges
   (b) converges
   (c) converges
   (d) diverges
   (e) converges
   (f) converges
   (g) converges
   (h) converges
   (i) diverges
   (j) diverges
   (k) diverges
3. Answers for section 3
   (a) converges
   (b) converges
   (c) diverges
   (d) diverges
   (e) converges
   (f) converges
   (g) converges
   (h) diverges
   (i) converges
   (j) converges
4. Answers for section 4
   (a) converges conditionally
   (b) converges absolutely
   (c) diverges
   (d) converges absolutely
   (e) converges consitionally
   (f) converges conditionally