Pay close attention to the language. Pay close attention to the structure of each function and systematically apply the rules! These are drill type problems, perfect for practice! Find more in your text-book!

I. Differentiate the following once (i.e. find the first derivative of each function)

1. $y = x^7$
2. $f(x) = x^7 + \frac{1}{8}x^8$
3. $g(x) = 16(x^7 + \frac{1}{8}x^8)$
4. $g(x) = \frac{1}{16}(x^7 + \frac{1}{8}x^8)$
5. $h(x) = \frac{1}{16}(x^7 + \frac{1}{8}x^8)^{16}$
6. $y = \frac{1}{(x^7+\frac{1}{8}x^8)^{16}}$
7. $y = \frac{x}{(x^7+\frac{1}{8}x^8)^{16}}$
8. $y = \frac{x^7}{(x^7+\frac{1}{8}x^8)^{16}}$
9. $y = \sin(3x)$
10. $y = \sin(3x^2 + 1)$
11. $y = \frac{\sin(3x^2+1)+2x^3-e^x}{\ln(2x^3+3\sin\sqrt{x})}$

II. Assuming that y is defined implicitly to be a function of x, find y'

1. $x^3y^3 + x^2y + 2x = 12$
2. $x^4 + y^2 + y^3 = 1$
3. $2x^3 - 3xy^4 + 5xy - 10 = 0$
4. $(x + y)^2 = 2x$
5. $\sqrt{1 + xy} - xy = 15$
6. $\frac{x}{x+y} - \frac{y}{x} = 4$

III. Find $\frac{d^2y}{dx^2}$ when y is a function of x (by now you should be assuming that it is implicitly)

1. $x^2 + y^3 + y = 1$
2. $y^2 + 2y = 5x$

IV. Find $\frac{dy}{dx}$ at $x = 1$ when $x^3y+xy^3 = 2$.

V. Prove that the second derivative, $\frac{d^2y}{dx^2}$, of $x^2 + y^2 = 1$ is never 0.

VI. Find $\frac{dy}{dx}$ in each of the following

1. $2\sin y + 3\cos x = 1$
2. $x\cos y - y\cos x = 3$
3. $4\sin^2 x - 3\cos^3 y = 1$
4. $\tan(x + y) = y$
5. $x + \sec xy = 5$
6. $x^3y + \tan^2 y = 3x$
7. $x = y^3 \csc^3 y$
8. $y = \cos(\tan x)$
9. $y = x^3 - x^2\cos x + 2x\sin x + 2\cos x$
10. $y = u^3 \sec u$, and $u = x\tan(x+1)$
11. $y = \sqrt[5]{3 - \sec v}$, and $v = \tan\sqrt{x}$
12. $y = \sqrt[3]{t^3 + 1}$, and $t = \sin\sin x$
13. $y = (1 + \sec^3 u)^{\frac{1}{3}}$, and $u = \sqrt{1 + \cos x^2}$

VII. Evaluate the limits.

1. $\lim_{x\to 0} \frac{\tan x}{x}$
2. $\lim_{x\to 0} \frac{\sin 2x}{x}$
3. $\lim_{x\to 0} \frac{(x+1)^2\sin x}{3x^3}$
4. $\lim_{x\to 0} \frac{1-\cos x}{x}$
5. $\lim_{x\to\infty} \frac{\sin\frac{2}{x}}{\sin\frac{1}{x}}$
6. $\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{(x-\frac{\pi}{2})^2}$

VIII. Differentiate!

1. $y = 3^{2x}$
2. $y = \log_{10}(2x+1)$
3. $y = e^{2\ln x}$
4. $y = \ln(\sin x)$
5. $y = x\ln(x+1)$
6. $y = \ln(\ln x)$
7. $y = \frac{e^{1-x}}{1-x}$
8. $y = e^{-2x}\sin 3x$
9. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$
10. $y = e^{-x}\ln x$
11. $y = x[\sin(\ln x) - \cos(\ln x)]$
12. $y = e^{\sin u}$, and $u = e^{\frac{1}{x}}$
13. $y = \ln(\cos v)$, and $v = \sin^2 x$
14. $y = \ln(x+y) + x^2 y$
15. $xe^y + x^2\ln y + y\sin x = 0$
16. $y = \ln(\sec x + \tan x)$
17. $y = x\sqrt{x^2+1} - \ln(x + \sqrt{x^2+1})$
18. $y = \ln(x + 4 + \sqrt{8x + x^2})$
19. $y = x - \frac{1}{4}\ln(1 + 5e^{4x})$
20. $e^{xy} = (x+y)^2$
21. $e^{\frac{1}{x}} + e^{\frac{1}{y}} = \frac{1}{x} + \frac{1}{y}$

IX. For $f(x)$ in each of the following, prove that the inverse, $f^{-1}(x)$ exists, graph it, and, at the specified point b, find the derivative of the inverse, $f^{-1'}(b)$.

1. $f(x) = 2x + 3\,;\, b = 3$
2. $f(x) = \sqrt{x+1}\,;\, b = 1$
3. $f(x) = x^2 + x\,;\, b = 0$
4. $f(x) = \frac{x+5}{2x+4}\,;\, b = \frac{5}{4}$
5. $f(x) = \frac{1}{x}\,;\, b = 1$
6. $f(x) = 3x^3 + 2\,;\, b = 2$
7. $f(x) = \sqrt{4 - x^2}$ when $0 \le x \le 2\,;\, b = \sqrt{3}$
8. $f(x) = 2x + |x|\,;\, b = 0$
9. $f(x) = x + |x|\,;\, b = 0$
10. $f(x) = x^4 + 2x^2 + 2$ when $x \le 0\,;$ $b = 2$
11. $f(x) = x^2 - 2x + 4$ when $x \ge 1\,;$ $b = 4$
12. $f(x) = \left(\frac{x+2}{x-2}\right)^3\,;\, b = -1$
13. $f(x) = \frac{x}{3+x^2}\,;\, b = \frac{1}{4}$

Answers!

I.

1. $7x^6$
2. $7x^6 + x^7$
3. $112x^6 + 16x^7$
4. $\frac{7}{16}x^6 + \frac{1}{16}x^7$
5. $(x^7 + \frac{1}{8}x^8)^1 5(7x^6 + x^7)$
6. $-16\,\frac{7\,x^6+x^7}{(x^7+1/8\,x^8)^{17}}$
7. $7\,\frac{x^6}{(x^7+1/8\,x^8)^{16}} - 16\,\frac{x^7\left(7\,x^6+x^7\right)}{(x^7+1/8\,x^8)^{17}}$
8. $3\cos 3x$
9. $6x\cos(3x^2+1)$

II.

1. $-\frac{2+2xy+3x^2y^3}{3x^3y^2+x^2}$
2. $\frac{-4x^3}{2y+3y^2}$
3. $\frac{3y^4-5y-6x^2}{5x-12xy^3}$
4. $\frac{1}{x+y} - 1$
5. $\frac{-y}{x}$
6. $\frac{2x^2+2xy-y^2}{x(2x+y)}$

III.

1. $\frac{-6y^2-2+12xyy'}{(3y^2+1)^2}$
2. $\frac{-25}{4}\,\frac{1}{(y+1)^3}$

IV. $\frac{1}{2}$

VI.

1. $3\,x^2\,\tan^3(x+1)$
   $\sec(x\,\tan(x+1))$
   $+\,3\,x^3\,\tan^2(x+1)$
   $\sec(x\,\tan(x+1))\,(\sec^2(x+1))$
   $+\,x^3\,\tan^3(x+1)$
   $\sec(x\,\tan(x+1))$
   $\tan(x\,\tan(x+1))\,(\tan(x+1)$
   $+\,x\,(\sec^2(x+1)))$
2. $-\frac{1}{10}\,\frac{\sec(\tan\sqrt{x})\,\tan(\tan\sqrt{x})\,\sec^2\sqrt{x}}{\sqrt{x}\,(3-\sec(\tan\sqrt{x}))^{(4/5)}}$
3. $\frac{\sin^2(\sin x)\,\cos(\sin x)\,\cos x}{(\sin^3(\sin x)+1)^{(2/3)}}$
4. $-\frac{x\sin(x^2)\sec^3(\sqrt{1+\cos x^2})\,\tan(\sqrt{1+\cos x^2})}{(1+\sec^3(\sqrt{1+\cos x^2}))^{(2/3)}\,\sqrt{1+\cos x^2}}$

VII.

1. 1
2. 2
3. ∞
4. 0
5. 2
6. ∞

VIII.

1. $(2\ln 3)\,3^{2x}$
2. $\frac{2}{\ln 10}\,\frac{1}{2x+1}$
3. $2x$
4. $\cot x$
5. $\ln(x+1) + \frac{x}{x+1}$
6. $\frac{1}{x\ln x}$
7. $\frac{xe^{1-x}}{(-1+x)^2}$
8. $e^{2\,x}\,(2\,\sin(3\,x) + 3\,\cos(3\,x))$
9. $4\,\frac{e^{-2\,x}}{(1+e^{-2\,x})^2}$
10. $-\frac{e^{-x}(\ln(x)x-1)}{x}$
11. $2\,\sin(\ln(x))$
12. $-\frac{\cos(e^{x^{-1}})e^{x^{-1}}e^{\sin(e^{x^{-1}})}}{x^2}$

13. $-2\,\frac{\sin(\left(\sin^2 x\right))\sin(x)\cos(x)}{\cos((\sin^2 x))}$

14. $\frac{\frac{1}{x+y}+2xy}{1-\frac{1}{x+y}-x^2}$

15. $-\frac{y\cos x+e^y+2x\ln y}{xe^y+x^2/y+\sin x}$

16. $\sec x$

17. $2\frac{x^2}{\sqrt{x^2+1}}$

18. $\frac{1+\frac{4+x}{\sqrt{8x+x^2}}}{x+4+\sqrt{8x+x^2}}$

19. $1-5\frac{e^{4x}}{1+5e^{4x}}$

20. $-\frac{2x+2y-ye^{xy}}{2x+2y-xe^{xy}}$

21. $-\frac{y^2(1-e^{1/x})}{x^2(1-e^{1/y})}$

IX.

1. ${f^{-1}}'(3)=\frac{1}{2}$
2. ${f^{-1}}'(1)=2$
3. $y'>0$ when $x>\frac{-1}{2}$. ${f^{-1}}'(0)=1$
4. ${f^{-1}}'(\frac{5}{4})=\frac{-8}{3}$
5. ${f^{-1}}'(1)=-1$
6. ${f^{-1}}'(2)=\frac{1}{36}$
7. ${f^{-1}}'(\sqrt{3})=-\sqrt{3}$
8. ${f^{-1}}'(0)$ does not exist
9. ${f^{-1}}'(0)$ does not exist
10. ${f^{-1}}'(2)$ does not exist
11. ${f^{-1}}'(4)=1/2$
12. ${f^{-1}}'(-1)=-1/3$
13. ${f^{-1}}'(\frac{1}{4})=8$