Prove that

$$\lim_{x \to \frac{1}{3}} 6x^2 + x - 1 = 0.$$

Proof:
Our task is to show that, for each $\epsilon > 0$, we are able to construct a $\delta > 0$ for which, whenever x is in the domain of our function $(6x^2 + x - 1)$, $0 < |x - \frac{1}{3}| < \delta$ necessitates that $|6x^2 + x - 1| < \epsilon$.
Firstly, we should examine the expression $|6x^2 + x - 1| < \epsilon$. Observe that $6x^2 + x - 1 = (2x + 1)(3x - 1) = 3(2x + 1)(x - \frac{1}{3})$, and that $x - \frac{1}{3}$ is one of the factors. If δ were equal to 1, we would have $0 < |x - \frac{1}{3}| < 1$ or $-\frac{2}{3} < x < \frac{4}{3}$ from which we derive that $-\frac{1}{3} < 2x + 1 < \frac{11}{3}$ or $3|2x + 1| < 11$. Now, $3|2x + 1||x - \frac{1}{3}| < 11|x - \frac{1}{3}|$ and either $0 < |x - \frac{1}{3}| < 1$ or $0 < |x - \frac{1}{3}| < \frac{\epsilon}{11}$ yields the desired inequality. Thus, for any ϵ, we should choose $\delta = \min(1, \frac{\epsilon}{11})$.
Let us verify that this indeed works. We consider two cases: $0 < \epsilon < 11$, and $11 \leq \epsilon$. In the first case, we select $\delta = \frac{\epsilon}{11}$, and $3|x - \frac{1}{3}||2x + 1| < 11\frac{\epsilon}{11} = \epsilon$ or $|6x^2 + x - 1| < \epsilon$, our desired inequality. In the second case, we select $\delta = 1$, and $3|x - \frac{1}{3}||2x + 1| < 11(1) < \epsilon$ or $|6x^2 + x - 1| < \epsilon$, our desired inequality.
Thus, we have shown what we had intended to show, and the limit must hold.