

I think that these problems represent the fundamentals which you should know. This is not an exhaustive set of problems; i.e. not all things which you should know are tested here. Some of the problems might be difficult. Enjoy!

I. Solve these inequalities:

1. $4 - 2x \leq 5$
2. $1 + 2x^2 < 5$
3. $\frac{2x-1}{x+2} > 1$
4. $|3 - 5x| \leq 2$
5. $|4x^2 - 7| > 2$
6. $3x - 2 < 7$
7. $\frac{3}{x} > 5$
8. $x^2 - 4 \leq 5$
9. $|3x - 2| < 7$
10. $|2x^2 + 5| \leq 6$
11. $|2x + 6| \leq 5$
12. $|x^2 - 4| \geq 5$
13. $|x^2 + 4| \geq 5$
14. $\frac{3x+5}{4x-2} < 3$
15. $\frac{1}{x+2} - \frac{1}{x-2} < 5$
16. $|x^2 + 2x - 3| \leq 1$
17. $\frac{x+5}{x-3} < \frac{x+2}{x+1}$
18. $\frac{2x+3}{x-5} > x + 4$

II. Express the given angle in radians.

- | | |
|--------------|--------------|
| 30° | -300° |
| 321° | 60° |
| 765° | 135° |
| 72° | 213° |
| 270° | -90° |
| -120° | |

III. Express the given angle in degrees.

- | | |
|----------|-----------|
| $\pi/3$ | $-5\pi/6$ |
| -3.6 | $-5\pi/4$ |
| 1 | 11 |
| $3\pi/2$ | 8π |

IV. Evaluate:

- | | |
|---------------|----------------|
| $\sin 0$ | $\cos \pi/6$ |
| $\tan \pi/4$ | $\csc \pi/3$ |
| $\sec \pi/2$ | $\cot 2\pi/3$ |
| $\sin 3\pi/4$ | $\cos 5\pi/6$ |
| $\tan \pi$ | $\csc 7\pi/6$ |
| $\sec 5\pi/4$ | $\cot 4\pi/3$ |
| $\sin 3\pi/2$ | $\cos 5\pi/3$ |
| $\tan 7\pi/4$ | $\csc 11\pi/6$ |
| $\sec 2\pi$ | $\cot 6\pi$ |

V. Sketch the graph of each of the following functions:

1. $3 \sin x$
2. $\sin 2x$
3. $3 \sin 2x$
4. $3 \sin(x + \pi/4)$
5. $\sin(2x + \pi/4)$
6. $3 \sin(2x + \pi/4)$
7. $3 \sin(2x + \pi/4) + 8$

VI. Solve for x.

1. $\sin x = \sqrt{3}/2$
2. $\cos x = 2$
3. $3 \sin x = \cos x$
4. $3 \cos^2 x = 1$

5. $\sin 2x = \sin x$

6. $\sin^2 x + 3 \sin x + 1 = 0$

7. $\sin^2 x + \sin x - 1 = 0$

8. $\sin x + \cos x = 1$

8. $\log_{10}(x(x-3)) = 1$

9. $2 \ln x + \ln(x-1) = 2$

10. $\log_a(x+2) + \log_a x = 2$

11. $\log_a(x(x+2)) = 2$

VII. Simplify:

1. $3^{\log_3(x^2)}$

2. $10^{-4 \log_{10} x}$

3. $\log_a(a^{-x+3})$

4. $\log_2 8 + \log_3 \frac{1}{27}$

X. Sketch:

1. e^{-x^2}

2. $\ln|x-1|$

3. xe^{-x}

VIII. Solve:

1. $\log_5 x = -3$

2. $\log_{10} x + \log_{10}(x+1) = 0$

3. $10^x - 12 + 10^{-x} = 0$

XI. Evaluate each of the following:

1. $\arctan -1/3$

2. $\arcsin 1/4$

3. $\arcsin(\sin \pi/2)$

4. $\sin(\arctan \sqrt{3})$

5. $\sin(\arcsin \frac{1}{\sqrt{2}})$

6. $\tan(\arcsin 3)$

7. $\arctan(\sin 1/6)$

8. $\arcsin(\sin 3\pi/4)$

IX. Find all real values of x which satisfy the given equation.

1. $\log_{10}(x+2) = -1$

2. $10^{3x} = 5$

3. $\log_{10}(x^2 + 2x + 1) = 1$

4. $\ln(x^2 + 2x + 10) = 1$

5. $10^{5-x^2} = 100$

6. $10^{1-x^2} = 100$

7. $\log_{10}(x-3) + \log_{10} x = 1$

XII. Solve:

1. $\sec(\sin x) = -\sqrt{2}$

2. $\cos(\arcsin x) = 1/2$

3. $\sec(\tan x) = -\sqrt{2}$

Answers!

I.

1. $[-1/2, \infty]$
2. $(-\sqrt{2}, \sqrt{2})$
3. $(-\infty, -2) \cup (3, \infty)$
4. $[\frac{1}{5}, 1]$
5. $(-\infty, -3/2) \cup (3/2, \infty)$
 $\cup (-1/2\sqrt{5}) \cup (1/2\sqrt{5})$
6. $(-\infty, 3)$
7. $(0, 3/5)$
8. $[-3, 3]$
9. $(-5/3, 3)$
10. $[-\frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{2}]$
11. $[-11/2, -1/2]$
12. $[-\infty, -3] \cup [3, \infty]$
13. $[-\infty, -1] \cup [1, \infty]$
14. $[-\infty, 1/2) \cup (11/9, \infty)$
15. $(-\infty, -2)$
 $\cup (-\frac{4}{3}\sqrt{5}, \frac{4}{3}\sqrt{5}) \cup (2, \infty)$
16. $[-1 - \sqrt{5}, -1 - \sqrt{3}]$
 $\cup [-1 + \sqrt{3}, \sqrt{5} - 1]$
17. $(-\infty, -11/7) \cup (-1, 3)$
18. $(-\infty, \frac{3}{2} - \frac{1}{2}\sqrt{101})$
 $\cup (5, \frac{3}{2} + \frac{1}{2}\sqrt{101})$

II.

$$\begin{array}{r} \frac{\pi}{6} \\ \approx 5.6025 \\ \frac{17\pi}{4} \\ \frac{2\pi}{5} \\ 3\pi/2 \\ -2\pi/3 \end{array} \quad \begin{array}{r} \frac{5\pi}{3} \\ \frac{\pi}{3} \\ \frac{3\pi}{4} \\ \approx 3.7176 \\ -\pi/2 \end{array}$$

III.

$$\begin{array}{r} 60 \\ \approx -206.2648 \\ \approx 57.2958 \\ 270 \end{array} \quad \begin{array}{r} -150 \\ -225 \\ \approx 630.2536 \\ 1440 \end{array}$$

IV.

$$\begin{array}{r} 0 \\ 1 \\ \text{not defined} \\ 1/\sqrt{2} \\ 0 \\ -\sqrt{2} \\ -1 \\ -1 \\ 1 \end{array} \quad \begin{array}{r} \sqrt{3}/2 \\ 2/\sqrt{3} \\ -1/\sqrt{3} \\ -\sqrt{3}/2 \\ -2 \\ 1/\sqrt{3} \\ 1/2 \\ -2 \\ \text{not defined} \end{array}$$

VI.

1. $\pi/3$
2. $x \notin \mathbb{R}$
3. $\arctan \frac{1}{3} + \pi k \quad (k \in \mathbb{Z})$
4. $\pm \arccos \frac{1}{\sqrt{3}} + \pi k \quad (k \in \mathbb{Z})$
5. $\pi k \quad (k \in \mathbb{Z})$
6. $\arcsin \left(\frac{-3 \pm \sqrt{5}}{2} \right) + \pi k \quad (k \in \mathbb{Z})$
7. $\arcsin \left(\frac{-1 \pm \sqrt{5}}{2} \right) + \pi k \quad (k \in \mathbb{Z})$
8. $2\pi k, \pi/2 + 2\pi k \quad (k \in \mathbb{Z})$

VII.

1. x^2

2. x^{-4}
3. $-x + 3$
4. 0

VIII.

1. $1/125$
2. $\frac{-1+\sqrt{5}}{2}$
3. $\log_{10}(6 \pm \sqrt{35})$

IX.

1. $-19/10$
2. $\log_{10} \sqrt[3]{5}$
3. $-1 \pm \sqrt{10}$
4. $x \notin \mathbb{R}$
5. $\pm\sqrt{3}$
6. $x \notin \mathbb{R}$
7. 5
8. 5

9. ≈ 2.3444
10. $x \notin \mathbb{R}$
11. $x \notin \mathbb{R}$

XI.

1. ≈ 0.3218 radians
2. ≈ 0.2527 radians
3. $\pi/2$
4. $\sqrt{3}/2$
5. $1/\sqrt{2}$
6. not defined
7. ≈ 0.1644 radians
8. $-\pi/2$

XII.

1. $x \notin \mathbb{R}$
2. $\pm\sqrt{3}/2$
3. ≈ 1.1694 radians, or ≈ 1.3214 radians